Radiant Energy  $Q$  [J]
The amount of energy carried by electromagnetic radiation.

Radiant Flux  $\Phi = \frac{dQ}{dt}$  [W]
The time rate at which radiant energy passes through a surface.

Flux Density  $F = \frac{d\Phi}{dA}$  [W m$^{-2}$]
Radiant flux per unit area intercepted by a surface.

Intensity of a point source  $I = \frac{d\Phi}{d\Omega}$  [W sr$^{-1}$]
Radiant flux emitted from a point source per unit solid angle. The intensity is, in general, a function of the direction of the radiant flux propagation ($\theta, \phi$).

Irradiance  $E = \frac{d\Phi}{dA}$  [W m$^{-2}$]
Flux density incident onto a surface.

Exitance, Emittance  $M = \frac{d\Phi}{dA}$  [W m$^{-2}$]
Flux density emerging out of a surface.
Radiance \[ L = \frac{d^2 \Phi}{|\cos \theta| \sin \theta \, dA \, d\Omega} \text{ [W m}^{-2} \text{ sr}^{-1}] \]

Flux density along a given direction \( \mathbf{s} \) passing through a unit area of a surface normal to the direction of propagation \( \mathbf{s} \) per unit solid angle. If the normal of the surface area of interest (\( dA \)) makes an angle \( \theta \) with the direction of propagation \( \mathbf{s} \), then the projected area \( |\cos \theta| \, dA \) is used in the calculation.

The radiance field \( L(x, y, z, \theta, \phi) \) is a function of the spatial location \( \mathbf{r} = (x, y, z) \) (the cartesian coordinates of the spatial location) and the direction of radiant flux propagation \( \mathbf{s} = (\theta, \phi) \) (the angular coordinates of the propagation vector).

Relation between radiance and irradiance

Emittance: \[ M = \int_{\text{outer half hemisphere}} L(\theta, \phi)|\cos \theta| \sin \theta \, d\Omega = \int_0^{\pi/2} \int_0^{2\pi} L(\theta, \phi)|\cos \theta| \sin \theta \, d\theta \, d\phi \]

Irradiance: \[ E = \int_{\text{inner half hemisphere}} L(\theta, \phi)|\cos \theta| \sin \theta \, d\Omega = \int_0^{\pi/2} \int_0^{2\pi} L(\theta, \phi)|\cos \theta| \sin \theta \, d\theta \, d\phi \]

Note: In the above equations, the outward normal of the surface of interest is taken as the \( z \)-axis (the upward direction if the surface is horizontal). For emittance calculation, all the flux emerging outwards is considered, i.e. all flux with \( \theta \) ranging from 0 (normal to surface, along \( z \)-direction) to \( \pi/2 \) (parallel to surface). For irradiance calculation, the flux going into the surface has \( \theta \) ranging from \( \pi/2 \) (parallel to surface) to \( \pi \) (along the \(-z\) axis, into the surface).
Example: Irradiance of a parallel beam

The radiance of a parallel beam of sunlight illuminating a horizontal plane can be modeled by,

\[ L = F \delta(\cos \theta + \cos \theta_s) \delta(\phi - \phi_s - \pi) = F \delta(u + u_s) \delta(\phi - \phi_s - \pi) \]

where \( F \) is the flux density of the solar beam (W m\(^{-2}\) through a surface normal to the beam), \( \theta_s \) is the solar zenith angle, and \( \phi_s \) is the solar azimuthal angle.

The irradiance on a horizontal plane is:

\[
E = \frac{2 \pi}{2} \int_0^\frac{\pi}{2} \int_0^\pi L(\theta, \phi) \cos \theta \sin \theta \, d\phi \, d\theta = \frac{2 \pi}{2} \int_0^\pi \int_0^1 L(\theta, \phi) |u| \, du \, d\phi = Fu_s; \text{ where } u_s = \cos \theta_s
\]

Example: Irradiance of an isotropic diffused radiance distribution

Suppose that the radiance field illuminating a horizontal surface is isotropic, i.e. independent of direction of propagation, i.e. \( L(\theta, \phi) = \text{constant} \). The irradiance on the surface is:

\[
E = \int_0^{\frac{\pi}{2}} \int_0^{\pi} L|\cos \theta| \sin \theta \, d\theta \, d\phi = \frac{2 \pi}{2} \int_0^\pi \int_0^1 |u| \, du \, d\phi = \pi L
\]
Example: A non-isotropic radiance distribution

Consider a surface emitting radiation with a radiance field:
\[ L(\theta, \phi) = L_0 \cos \theta \]

The emittance of the surface is
\[ M = \frac{\pi}{2} \int_0^\pi \int_0^{2\pi} L(\theta, \phi) \cos \theta | \sin \theta \, d\theta \, d\phi = L_0 \int_0^2 \int_0^1 u^2 \, du \, d\phi = \frac{2\pi}{3} L_0 \]

Spectral Quantities

So far, the radiant quantities considered above are broadband quantities, i.e. quantities integrated over a wide wavelength band. The distribution of a given radiant quantity within a wavelength band is characterized by the respective spectral quantities. If \( X \) is a wideband radiant quantity, then the corresponding spectral quantity is often denoted by \( X_\lambda \). They are related by
\[ X(\lambda_1 \text{ to } \lambda_2) = \int_{\lambda_1}^{\lambda_2} X_\lambda \, d\lambda \]

Thus, we have the following spectral radiant quantities:

- **Spectral radiant energy**, \( Q_\lambda \) [J \( \mu \text{m}^{-1} \)]
- **Spectral radiant flux**, \( \Phi_\lambda \) [W \( \mu \text{m}^{-1} \)]
- **Spectral flux density**, \( F_\lambda \) [W m\(^2\) \( \mu \text{m}^{-1} \)]
- **Spectral intensity**, \( I_\lambda \) [W sr\(^{-1}\) \( \mu \text{m}^{-1} \)]
- **Spectral irradiance**, \( E_\lambda \) [W m\(^2\) \( \mu \text{m}^{-1} \)]
- **Spectral exitance**, Spectral emittance, \( M_\lambda \) [W m\(^2\) \( \mu \text{m}^{-1} \)]
- **Spectral radiance**, \( L_\lambda \) [W m\(^2\) sr\(^{-1}\) \( \mu \text{m}^{-1} \)]